Inverse analysis of biomagnetic fields in consideration of the external magnetic noise

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1 Introduction

Even in magnetic shield, external magnetic field could not be neglected especially for magneto-meter system, and, it will be considered as uniform external noise. Also, the noise is caused by nearer external source, and is not uniform one of the noise. In this paper, the influence of the external uniform field and the external gradient noise on inverse problem solution by the SMN (Selective Minimum-Norm) method is studied by simulation.

2 Method

2.1 Conventional SMN method

Selective Minimum-Norm (SMN) method is used for the estimation of the biomagnetic source dipoles. The SMN method has following 5 characters:

1. Calculation time is realistic.
2. It can get the sparse solutions.
3. It can treat the magnetic field source that the number of the component is unknown.
4. It can be applied to the three-dimensional model that is not limited with the foresight information.
5. It can manage the data with some noise.

By the above characters, the SMN method is very useful for the estimation of the biomagnetic source dipoles[1].

According to the Biot-Savart law, the measured magnetic field $\boldsymbol{b} = (b_1, b_2, \cdots, b_M)^T$ is related to the current dipole moments $\boldsymbol{q} = (q_1, q_2, \cdots, q_{2N})^T$ by the following equation:

$$\boldsymbol{Aq} = \boldsymbol{b} \tag{1}$$

where the coefficient matrix $\boldsymbol{A}$ is computed by Biot-Savart law and is called lead-field matrix, and every dipole has two degree of freedoms, θ and ϕ.

In ordinary cases, N, the number of the grid point is much greater than M, the number of sensor (in this study, N=16776, 16777, or 16778, M=57). The column vector of the lead-field matrix is normalized to handle all moments equally. In order to satisfy Eq. (1), we select M variables and force the other variables to zero. Then, we can obtain ${}_{2N}C_M$ feasible solutions. In order to extract a best solution from feasible solutions, we minimize an L_1-norm of vector by selecting location of moments. So we can use simplex Linear Programming for finding the best solution [2].

2.2 Extended SMN method

In section 2.1, we showed that SMN method can cope with some noise. However, non-existent current dipoles are sometimes estimated with the conventional SMN method. In this section, we discuss the principle of the extended SMN method that can estimate the external uniform magnetic field in z direction.

In order to take the uniform magnetic field into consideration, we will add $\boldsymbol{D}$ to $\boldsymbol{A}$, and $\boldsymbol{p}$ to $\boldsymbol{q}$, then we obtain:

$$\begin{pmatrix} a_{1,1} & \cdots & a_{1,2N} & d_{1,1} & d_{1,2} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ a_{M-1,1} & \cdots & a_{M-1,2N} & d_{M-1,1} & d_{M-1,2} \\ a_{M,1} & \cdots & a_{M,2N} & d_{M,1} & d_{M,2} \end{pmatrix} \begin{pmatrix} q_1 \\ \vdots \\ q_{2N} \\ p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ b_{M-1} \\ b_M \end{pmatrix} \tag{2}$$

where the value $\boldsymbol{p}$ represents the uniform magnetic field strength. If you stick to the dipole moment, you can think $\boldsymbol{p}$ is a dipole placed in a very far distance on the x-y plane.

Using Eq. (2), we can get good solutions in consideration of the noise.

2.3 Condition for estimation

The model of a head is considered to be a hemi-spherical body (radius r=10[cm]). Arrange the surface of sensors and the surface of spherical body in parallel.

In this study, the number of the sensor M is 57[channel], the mesh grid is 5.0[mm], and the mesh points N is 16776 (in case of the extended SMN method, N=16777).

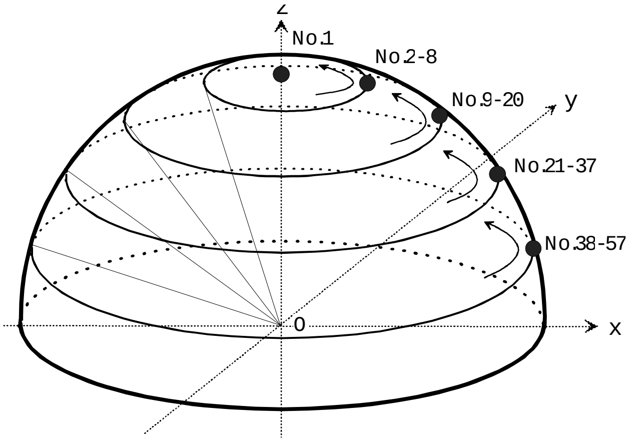


Figure 1. *Estimation range and potision of sensors.*

3 Result and discussion

3.1 Consideration of the uniform magnetic field

The magnetic field which is measured by sensors and the estimated solutions are shown in Fig. 2 (a), and (b) and (c) respectively. Here, the position of the primary current dipole is (5,0,0)[cm].

The size of the data circle is magnitude of the estimated current dipole, and the line from a circle origin is the moment's orientation.

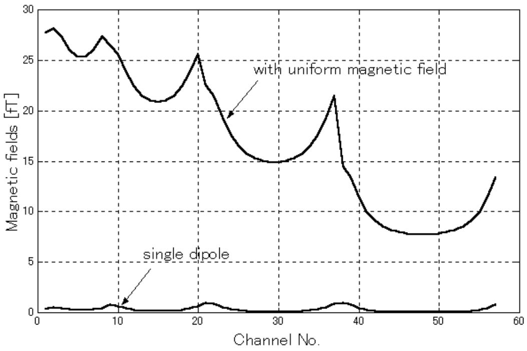
When the uniform magnetic field is added, it was found out that some primary current dipoles were estimated around the origin of the sphere by the simulation in the conventional SMN method.

This shows that some current dipoles which are around the origin of the sphere were selected as biomagnetic sources that make the uniform magnetic field. While, it is clear that the selected dipoles are nearly at (5,0,0)[cm].

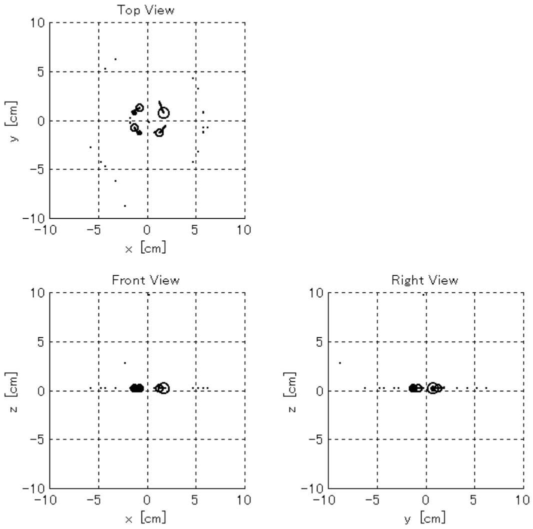
Hence, it is found out that the additional uniform magnetic field was estimated as moment ***p***. The result of single current dipole estimation were not different among the conventional and the extended method. Hence, there was no deterioration of the estimated precision in the extended SMN method in comparison with the conventional method.

The extended SMN method can achieve not only more precision but also shorter calculation time than the conventional method.

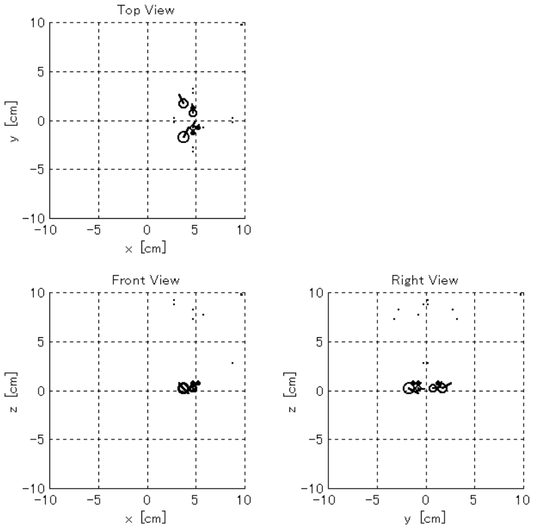
We will discuss calculation time in section 3.3.



(a) Magnitude of magnetic field at each sensor.



(b) Solution by the conventional SMN method.



(c) Solution by the extended SMN method.

Figure 2. *Consideration of the uniform magnetic field.Position of primary current dipole:(5,0,0)[cm].*

3.2 Consideration of the external gradient noise

In section 3.1, we discussed the estimation of biomagnetic source dipole in consideration of the uniform magnetic field. In this section, we will discuss the case of the external gradient noise.

In order to take uniform magnetic fields in x direction into consideration, we add a new current-dipole $\boldsymbol{p}$ further (N=16778).

Assume that the magnetic field noise source is at (50,0,0)[cm]. The magnetic field which measured by each sensors and the estimated solution are shown in Fig. 3(a), and (b) and (c) respectively. Here, the primary current dipole position is (5,5,5)[cm].

The position of the primary dipole could be estimated by the extended SMN method even from the magnetic field with noise. Furthermore, when the same amount of noise was added to each sensor, it could be estimated precisely and rapidly more than the conventional SMN method (Fig. 4.). However, when many noise sources are supposed, precise estimation cannot be done conversely, and it sometimes extends the calculation time.

It is because a part of the magnetic field of the primary current dipole is estimated as the magnetic field of noise by supposing many noise sources. Therefore, if the number of the parameter representing noise is increased, the estimation of the precise magnetic fields source cannot be always improved.

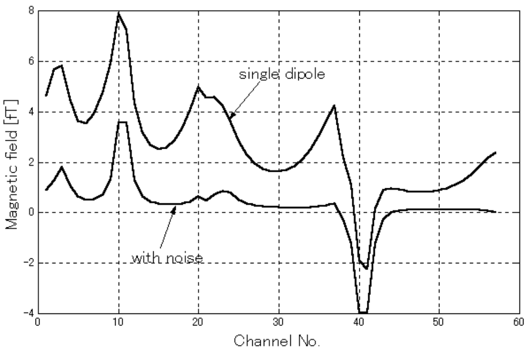
3.3 Calculation time

The calculation time by the conventional and the extended SMN method are shown in Table 1.

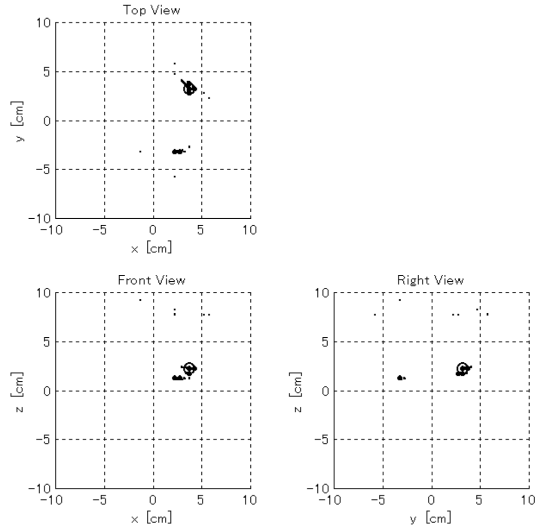
Table 1. *Compared calculation time by the conventional and the extended SMN method.*

	A	B	C	D
Conventional SMN-method	0’13”	3’30”	0’45”	3’47”
Extended SMN-method	0’13”	0’58”	0’36”	1’03”

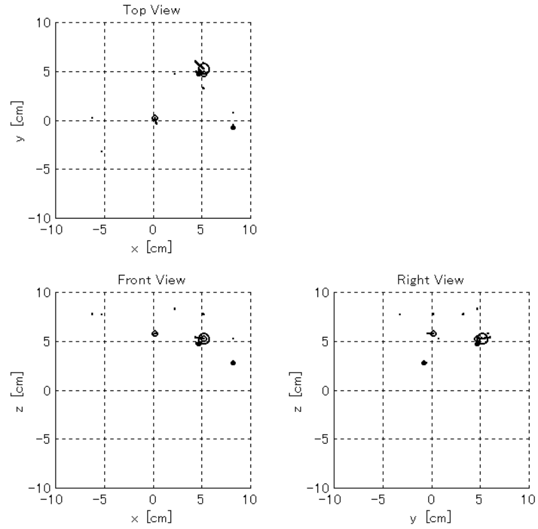
A: Single dipole is at (5,5,5)[cm]
B: Consideration the uniform magnetic field (Fig. 2).
C: Consideration the external gradient noise (Fig. 3).
D: Consideration of the same amount of noise (Fig. 4).
Used machine: CPU AMD Athlon, 950MHz



(a) Magnitude of magnetic field at each sensor.



(b) Solution by the conventional SMN method.



(c) Solution by the extended SMN method.

Figure 3. *Consideration of the external gradient noise. Position of primary current dipole: (5,5,5)[cm].*

Clearly, the calculation time by the extended SMN method is faster than the conventional method.

This is because it takes time to search for the biomagnetic source dipole that does not exist with the conventional SMN method. However, this calculation time varies according to the primary dipole position greatly.

Precision and a calculation time could be improved by using the extended SMN method.

4 Conclusion

We did the estimation of the biomagnetic source dipoles in consideration of the uniform magnetic field and external gradient noise by the simulation. It became possible that the position of the biomagnetic source dipole was estimated precisely by adding a number of the parameter of external magnetic fields in the conventional SMN method. Furthermore, the calculation time is shorter than before.

The external magnetic field can be canceled by the extended SMN method. However, it has the possibility that original data are injured in case of too many additional parameters. Therefore, it is necessary for the extended SMN method to add parameters that suit environment.

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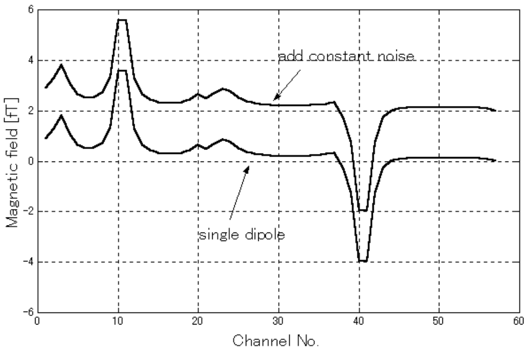
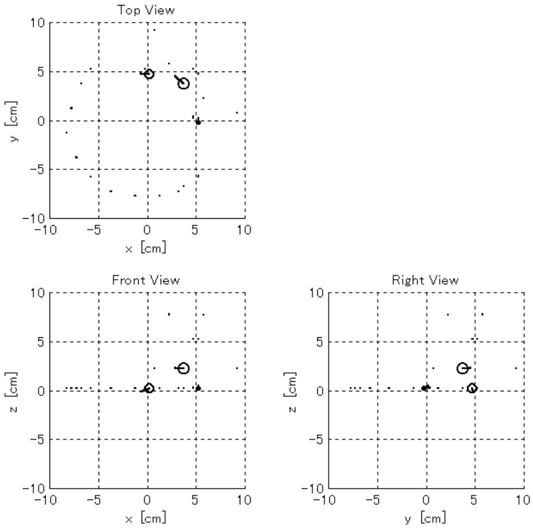


(a) Magnitude of magnetic field at each sensor.



(b) Solution by the conventional SMN method.

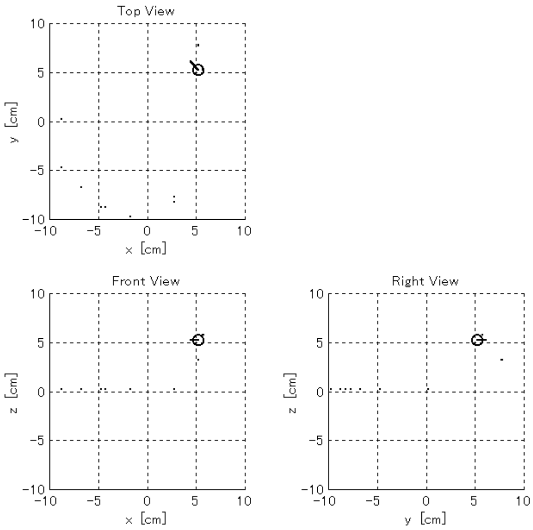


Figure 4. *Consideration of the same amount of noise. Position of primary current dipole:(5,5,5)[cm].*